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# Modeling of the LIFE minichamber Xe theta pinch experiment

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## ABSTRACT

The LIFE minichamber experiment will investigate cooling of the strongly radiating Xe buffer gas protecting the LIFE chamber wall. A theta pinch will inductively heat a few cc of Xe at ion density  $2 \times 10^{16}/\text{cc}$  to several eV. Thomson scattering will be used to determine electron temperature and ionization state. Modeling is being done using the magnetohydrodynamic code HYDRA with an external circuit model and inductive feedback from the plasma to the external circuit. Coil stresses are being assessed using the 3D MHD code ALE3D. A major challenge to the design is the paucity of opacity and conductivity data for Xe in the buffer gas regime. Results of the modeling will be presented.

**Keywords:** Fusion, Xe, atomic physics, theta pinch, MHD

## 1. INTRODUCTION

In the LIFE fusion chamber, Xe buffer gas having ion density  $1 \times 10^{16}$ - $4 \times 10^{16}/\text{cc}$  will protect the chamber wall from 5-20 MJ in prompt X-rays, which will ionize the Xe and heat it to an electron temperature  $T_e$  of 10 eV and higher. The Xe is expected to cool and recombine to below 1 eV and an ionization state  $Z^*$  near zero. It is necessary to know what  $T_e$  and  $Z^*$  will be between shots, because they affect propagation of drive beams and survival of the incoming cryogenic targets (entering the chamber at a 15 Hz repetition rate). At low  $T_e$  and  $Z^*$  there is a paucity of atomic data for Xe. Cooling by neutral collisions is important, and may be intractable to model. Xe radiates effectively, making it difficult to produce and study a hot homogeneous volume. In the LIFE minichamber experiment<sup>1</sup> a theta pinch will inductively heat a few cc of Xe to a few eV.  $T_e$  and  $Z^*$  will be measured as the Xe cools using Thomson scattering. Ion temperature may be measured. Modeling of the experiments is being done using the magnetohydrodynamic code HYDRA with an external circuit model and inductive feedback from the plasma to the circuit. Coil stresses are being assessed using the 3D MHD code ALE3D, to confirm that coil motion in response to the drive will not damage the tube containing the Xe, or the coil itself. The scarcity of Xe opacity and conductivity data in the buffer gas regime presents a challenge to design of the experiment. Preliminary results suggest that the behavior of the theta pinch circuit and the plasma is complex, and that the Xe may be heated to  $T_e$  of 2-3 eV before cooling begins.

### 1.1 Key issues in plasma and coil response

The modeling effort described here addresses the following questions. Do we understand the plasma response? Will the plasma get hot enough, and be homogeneous and settled enough to diagnose? The key results so far of a 1D cylindrical HYDRA MHD model are that  $T_e$  of 2-3 eV will be achieved before cooling, that the plasma response will be dynamic and complex, and that pinching will moderate the potential Joule heating.

In addition, it is important to assess whether coil stress from the pulse will damage the coil or the tube. 3D ALE3D MHD modeling suggests that peak von Mises stress below 30 Mpa will be reached, well below the level of concern of 100 Mpa. The modeling also suggests that static stress conditions not approached, so that the coil response is not described well by static models such as hoop stress.

Other issues that are being addressed in ongoing work include 2D end effects, plasma-specific effects (*eg.* the Hall term), 3D MHD instabilities, sensitivity to atomic models, and the cooling behavior out to 100 ms relevant to LIFE chamber conditions. Accurately modeling the late time cooling requires the data the experiment is intended to provide.

## 1.2 Challenges to experiment and modeling

There are a number of challenges to the experiment and the modeling of the experiment. Diagnostics will be limited initially, mainly Thomson scattering, which is ineffective as  $Z^*$  goes to zero. Ultimately the key measurement is ion temperature, which is hard to measure for a cool, nearly neutral gas. The design is bootstrapped: unknown Xe atomic properties determine the conductivity and emissive opacity. Xe radiates very effectively — making it difficult heat Xe and keep it hot until settled homogeneous conditions are achieved. Pinching delays creation of homogeneous conditions — in this paper we discuss approaches to mitigating the effects of pinching. The finite length of the coil causes hydro effects due to end waves generated at the interface between the heated and unheated Xe — this determines the length of drive coil needed. While the theta pinch plasma is theoretically stable to the ‘sausage’ mode<sup>2</sup>, it is potentially unstable to the ‘kink’<sup>2</sup> mode, which is a 3D mode and therefore challenging to model.

In addition, plasma-specific effects may matter during heating: a large plasma current is present during the Thomson scattering measurement, magnetic pressure dominates thermal pressure, and the Hall term<sup>3</sup> is large — this will cause radial charge separation, and is not currently modeled. Furthermore, key time scales are similar — on the order of 100 ns for the drive rise, diffusion of the drive magnetic field into the Xe, and ion-electron equilibration. NLTE is needed to model the dynamically evolving plasma.

## 1.3 Codes

The EM code Maxwell<sup>6</sup> is used to design external circuit and determine the nominal drive. The MHD radiation-hydro code HYDRA<sup>7</sup> is used to the 1D plasma response. HYDRA is a single fluid, multiblock, multimaterial ALE radiation hydrodynamics code. HYDRA solve the resistive MHD equations in the small Hall limit, and solves the radiative transfer equations. The Python interpreter is used with the HYDRA model to implement a circuit model including inductive feedback from the plasma to the coil. The MHD mechanical code ALE3D<sup>8</sup> is used to assess coil stresses. ALE3D is an arbitrary-Lagrangian-Eulerian (ALE) code that uses a hybrid finite element and finite volume formulation to model fluid and elastic-plastic response of materials on an unstructured grid.

## 1.4 Assumptions/simplifications

Simplifications in the current modeling include the following: the initial plasma breakdown mechanism (eg. an ionizing current applied directly to the Xe) is not modeled; scalar Spitzer conductivity or constant conductivity is used; heat conduction to the tube walls is assumed negligible.

## 1.5 Atomic models

The main atomic physics model used in HYDRA is the Detailed Configuration Accounting (DCA) model, which is a nonlocal thermal equilibrium (NLTE) model<sup>4</sup>. DCA includes collisional and photo excitation/ionization, autoionization, and inverse processes; charge exchange and neutral collisions are not included. The IOMIX model<sup>5</sup> has also been used; a comparison of DCA to IONMIX is presented in this paper. IONMIX can be run LTE, which assumes 3-body recombination collisions dominate all atomic processes, or NLTE (‘coronal’), where gas density is sufficiently low that 2-body radiative effects are the dominant mechanism of recombination and deexcitation.

Generally speaking, different  $T_e$  and  $T_i$  does not mean a system is in NLTE; it implies that the electrons and ions are not equilibrated but says nothing about the electron atomic states and the radiation field. By definition LTE is a balance between radiationless atomic processes, *ie.* 3 body recombination balances collisional ionization. This makes it possible to compute LTE opacities without knowing about the radiation field. If NLTE opacities are computed then either something is assumed about the radiation field or the radiation field is actually computed while opacities are computed the ‘in-line’. A crude rule of thumb formula is that if the electron density  $N_e$  is such that

$$N_e < 1.6e12 \times \sqrt{T_e} \times I_{mn}^{-3}, \quad (1)$$

where  $T_e$  is in eV,  $I_{mn}$  is the ionization energy in eV, and  $N_e$  is in units of  $\text{cm}^{-3}$ , then the system is in NLTE. Thus, hot low density plasmas are NLTE. Cool low density plasmas are in LTE. The condition of the Xe buffer plasma in the LIFE chamber and the theta pinch experiment is not obviously LTE for all temperatures.

The question of LTE vs. NLTE for the low-temperature Xe is complicated. The crude formula (1) only addresses whether electron collisional rates dominate photo recombination rates. It does not address timescales, which can become long when the free electrons disappear. Codes like DCA and FLYCHK<sup>9</sup> will not do a reasonable job on neutral Xe, as they were not designed for this regime. The detailed Scram model<sup>3</sup> should do better, but also does not incorporate collisions with neutrals, which may be required to achieve LTE. These are key reasons to perform the theta pinch experiment. In contrast, calculating the temperature at which the electrons and ions decouple is more straightforward, but still somewhat uncertain. 4000-8000 K is a rough estimate, but such uncertainty would be significant for LIFE target survival.

## 1.6 Previous work

Theta pinches have been used in fusion research<sup>10</sup>.

## 2. 1D MHD MODEL OF XE PLASMA RESPONSE

### 2.1 1D Model of the coil, tube and plasma

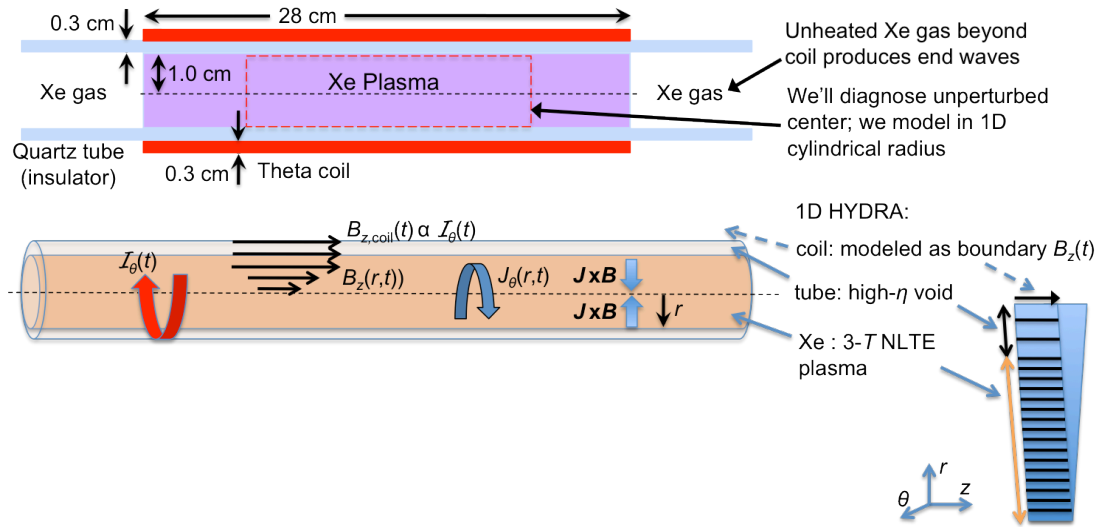
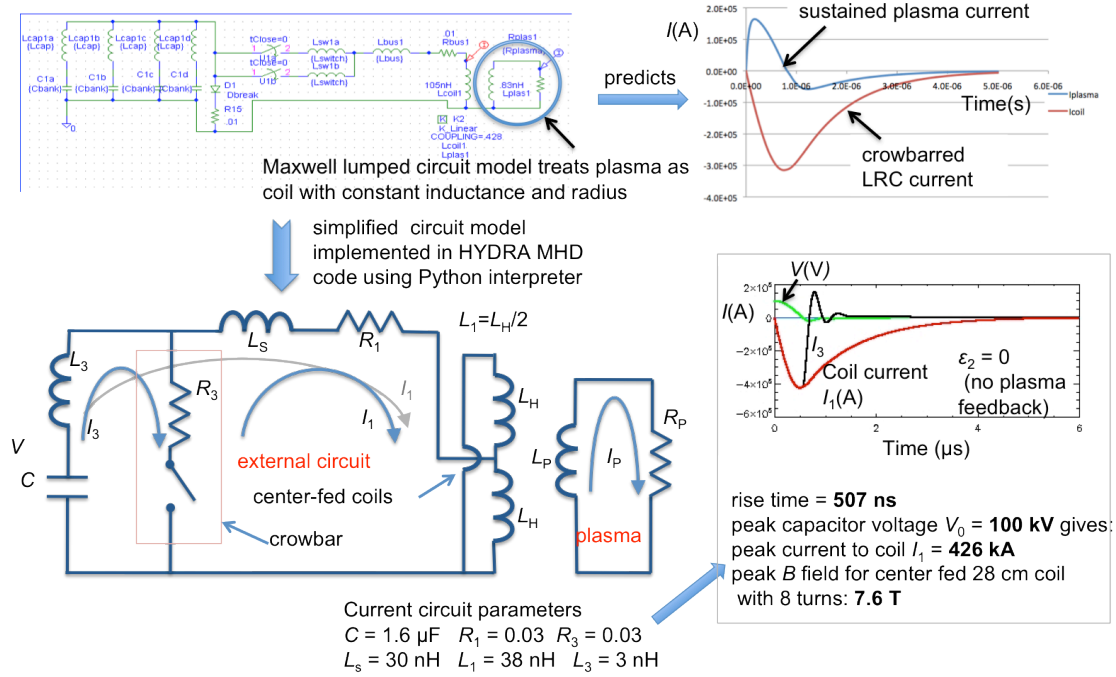


Figure 1. A simple 1D model of the theta pinch is coaxial infinite-length solenoids and low- $\beta$  plasma.

Figure 1 shows a schematic of the theta pinch coil, tube, and Xe plasma region, and a 1D cylindrical model of the plasma. Unheated Xe gas beyond the ends of the coil produces end waves that propagate into the Xe. The unperturbed center will be diagnosed; this part of the plasma is modeled in 1D cylindrical radius. The coil current generates a solenoidal magnetic field  $B_{z,coil}(t)$  parallel to the tube axis that diffuses rapidly through the insulating tube and slowly (100 ns time scale) into the Xe plasma. The resulting field  $B_z(r,t)$  has spatial gradient in the radial direction that is equivalent to an azimuthal current  $J_\theta(r,t)$ . In 1D HYDRA, the coil is modeled only as the boundary field  $B_{z,coil}(t)$ , the tube is a high-resistivity void, and the Xe is a three-temperature NLTE plasma.

## 2.2 Circuit model



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Figure 2. A simplified external circuit is modeled using Python.

Figure 2 shows the Maxwell lumped circuit model of theta pinch and plasma; the drive current and the plasma current predicted by the lumped circuit model; the simplified circuit model implemented in the the Python interpreter for use in the HYDRA MHD model; and the circuit voltages and currents predicted by the simplified model. The lumped circuit model predicts a typical crowbarred LRC drive current, and a sustained plasma current that oscillates once, reaching zero again at about the time of peak drive current. A simplified circuit model is implemented in the Python interpreter within HYDRA. This model enables modeling the effect of back EMF from the plasma to the coil. With representative circuit parameters, the drive rise time will be about 500 ns, and a peak capacitor voltage of 100 kV would give a peak drive current to the coil of about 400 ka, and a peak solenoidal  $B_{z,\text{coil}}$  field of about 8 Tesla for center fed, 8-turn coil 28 cm in length.

## 2.3 1D evolution of Xe plasma

Figure 3 shows the evolution of the Xe plasma in the 1D HYDRA MHD model. Each panel shows snapshot spatial profiles at the times indicated. The tube inner radius is 1.0 cm. The plots of  $B$  field show the coil field (right side) rising with time, and the field diffusing into the plasma. Due to pinching, the field lines at low radius are compressed, so that the field at the center exceeds the coil field at 400 ns; after that the field flattens out. The pinching is also evident in the plots of current  $J_\theta$ , which show a reversal of the current between 250 ns and 400 ns, with a intermediate-time flattening at 300 ns. The pinching is also evident in plots of radial velocity, which show a large inward-directed velocity of 2–3 cm/ $\mu\text{s}$  (2–3 km/s), and in plots of ion number density  $n_i$ , which show the outer part of the tube being evacuated and the density at low radius increasing by a factor of ten. The Joule heating, which is locally proportional to  $J_\theta^2$ , occurs in two

main pulses, reflecting the current reversal. Electron temperatures  $T_e$  of 2-3 eV are achieved, and the ionization state  $Z^*$  reaches three to four. These plots show the degree to which the plasma will be settled and homogeneous during the measurement; the Thomson scattering diagnostic will interrogate the plasma at low radius.

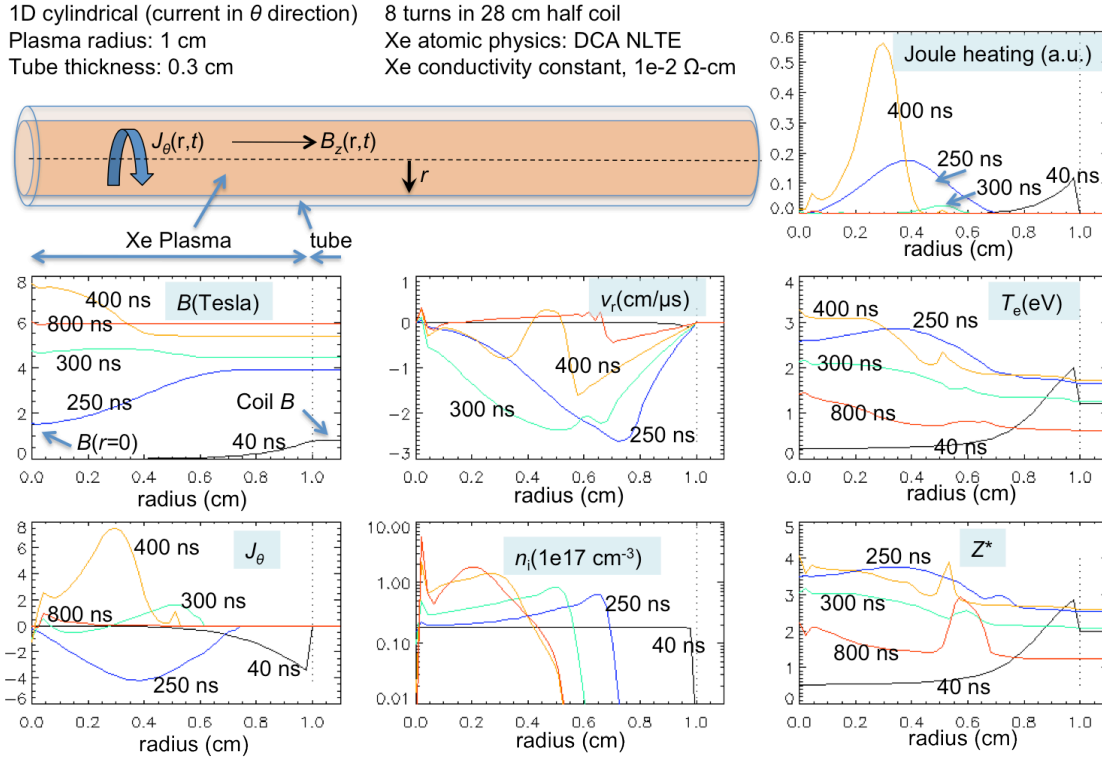


Figure 3. 1D HYDRA predicts significant pinching, and peak  $T_e$  of 2-3 eV.

## 2.4 Pinching

Due to pinching the total plasma current oscillates and attenuates much faster in the HYDRA model than in the lumped circuit model, as shown in Figure 4. The total plasma current is defined as the total current through an azimuthal face within the plasma, integrated over radius. The early oscillation is reflected in the predicted time histories of Joule heating,  $T_e$  and  $Z^*$ , where the latter two are mass-weighted over the Xe plasma. It is the compression of the  $B$  field at low radius that produces the early oscillation in the plasma current, as follows. It follows from Ampere's law that the total plasma current is proportional to the difference between the coil field (at the outer radius of the plasma), and the field at the center of the plasma. Therefore, when the field at the center exceeds the coil field, the plasma current reverses sign. Other numerical investigations confirm that the oscillation time of the plasma current is only weakly dependent upon the drive rise time, and is not changed by either omitting radiation in the model or by trapping all radiation within the model. The early oscillation appears to be a robust effect of pinching.

Pinching could be mitigated by using a larger inner tube (outer plasma) radius. A larger plasma radius would delay pinching by delaying compression of the  $B$  field at low radius within the plasma. Figure 5 shows a study of using larger radii, and the attendant Joule heating. Optimization of the tube radius to maximize Joule heating may be possible. However, a larger tube radius requires heating more plasma with the same drive energy, is likely to produce shocks within the plasma since waves have more space and time to steepen, and may lead to less homogeneous conditions for diagnosis.

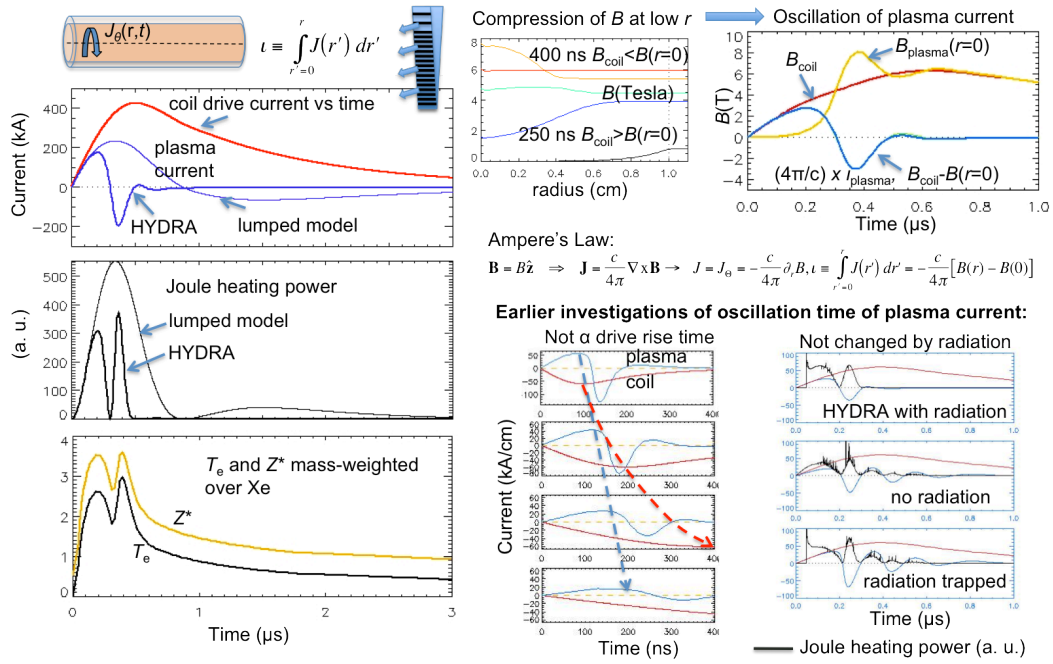


Figure 4. Pinching means total plasma current oscillates faster than lumped circuit model predicts, moderating the Joule heating.

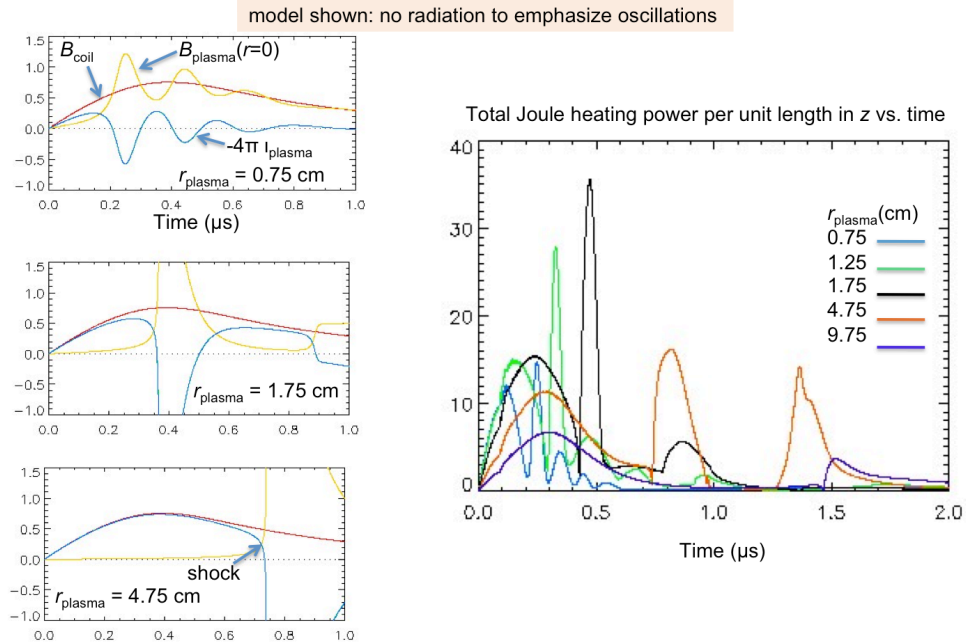


Figure 5. Larger tube radius delays pinching but requires heating more plasma, may shock the plasma, and delays homogeneity.



## 2.5 Background $B_{0z}$ field

A constant background magnetic  $B_{0z}$  field, produced by fixed magnets or a second, DC coil, could delay pinching and increase the time over which the plasma is heated. The  $B_{0z}$  field could (a) have the same direction as the drive field, which could potentially produce a larger magnetic pressure resisting compression, or (b) oppose the drive field, which would give a smaller net central field to compress. As Figure 6 shows, the opposed background field appears to be more effective, increasing the time over which the plasma is heated, but without, however, increasing the peak  $T_e$  and  $Z^*$ .

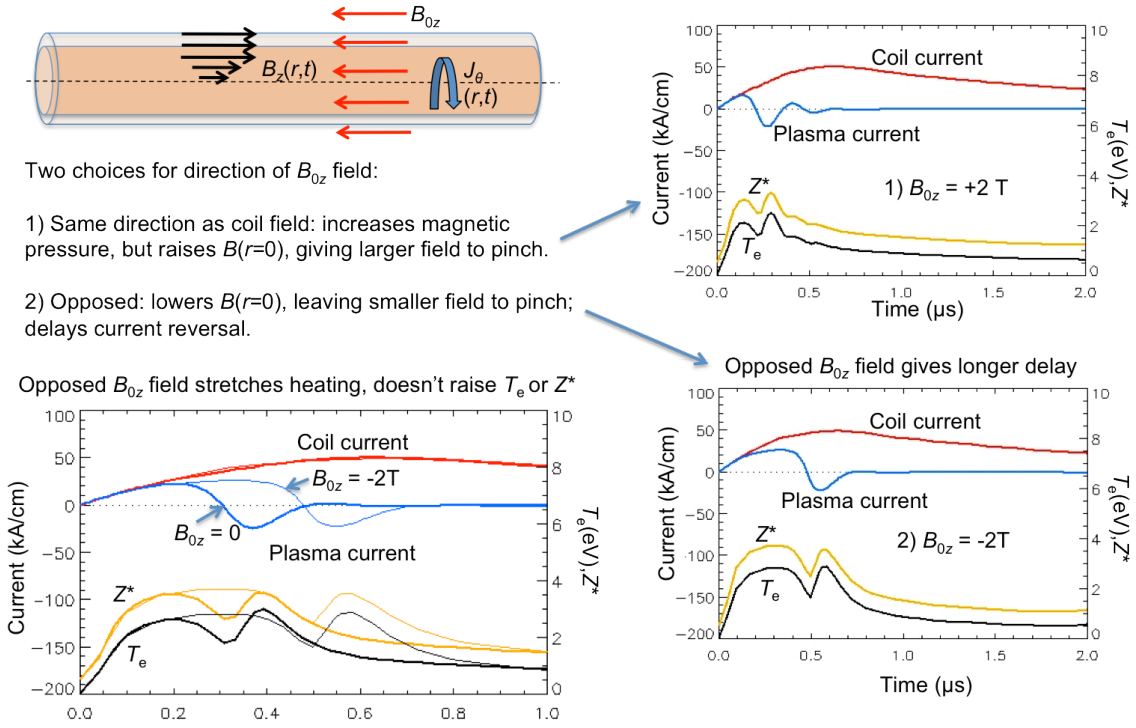


Figure 6. A constant background  $B_{0z}$  field produced by fixed magnets or 2<sup>nd</sup>, DC coil could delay pinching.

## 2.6 Back EMF

Back EMF from the plasma to the coil modifies the nominal drive at the 15-20% level when added to the circuit model. As Figure 7 shows, adding back EMF to the circuit model stretches the pulse, and lowers the currents,  $T_e$  and  $Z^*$ . The magnitude of the back EMF is significant compared to the magnitude of the voltage on the drive capacitor. Back EMF apparently does not produce a very large effect, but it is relatively straightforward to add to the circuit model.

## 2.7 Hall term electric field

The Hall term field, which is currently not modeled, would be comparable in magnitude to other terms in the electric field, so significant radial charge separation is expected. Figure 8 reprises plots of  $B$ ,  $J$ , and radial velocity near the time of current reversal. The lower panels show the magnitudes of the resulting terms in the  $E$  field, where  $\mathbf{v} \times \mathbf{B} + \eta \mathbf{J}$  is usual Ohm's law field, and  $\mathbf{J} \times \mathbf{B} / (n_e e)$  is Hall term field.

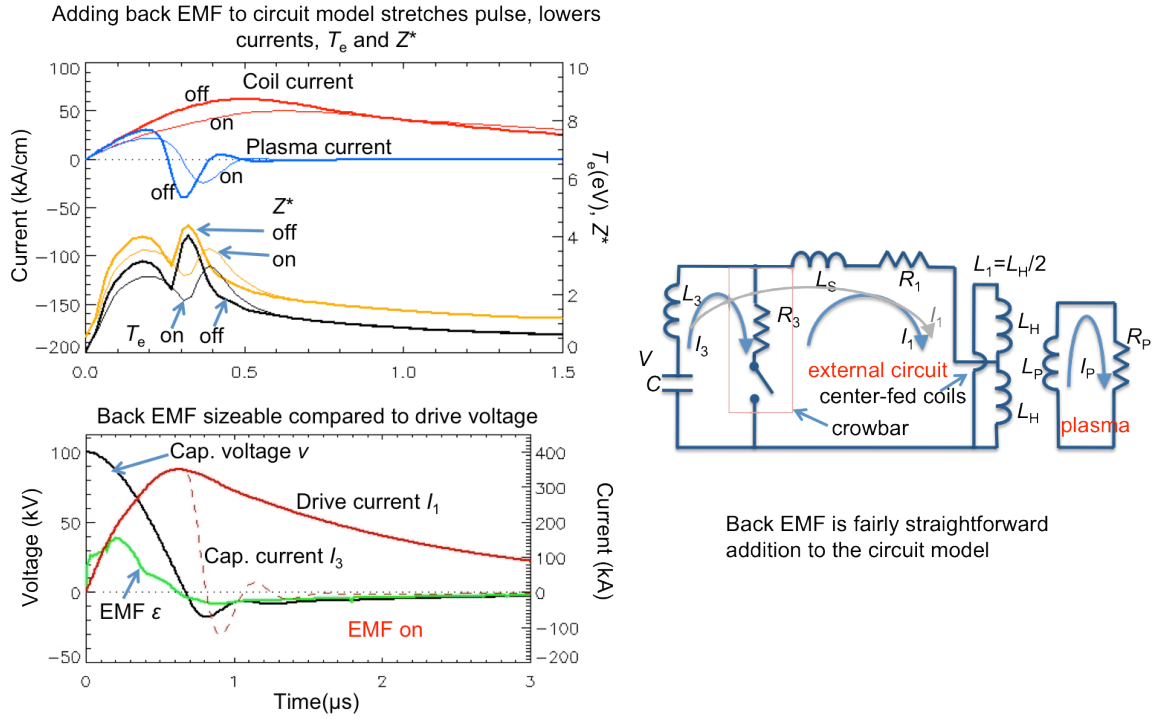


Figure 7. Back EMF from the plasma to the coil modifies the nominal drive at the 15-20% level when added to the circuit model.

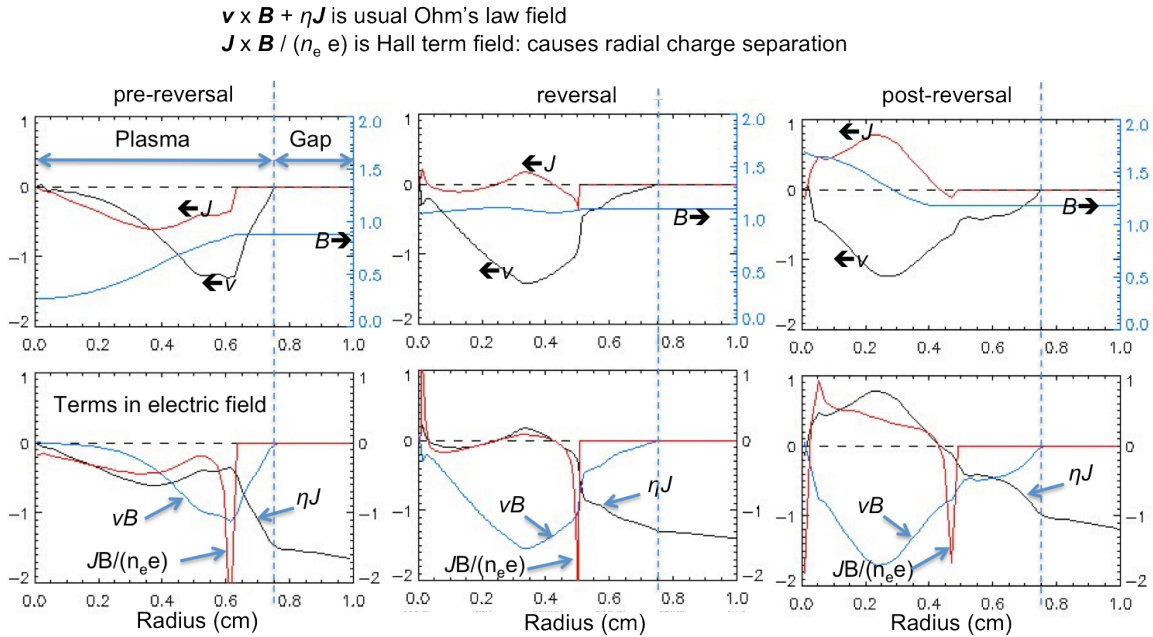


Figure 8. Hall  $\mathbf{J} \times \mathbf{B} / (n_e e)$  term in the electric field is significant; radial charge separation is likely.

## 2.8 Opacity models

Different opacity models produce different predictions for  $T_e$ . Figure 9 shows time histories for the DCA model and for the IONMIX model, which is currently being updated. IONMIX predicts a significantly higher  $T_e$ . Such a temperature uncertainty is significant for LIFE target survival. However, Xe atomic models are not tuned for this regime — we are usually extrapolating to low temperature and density, well below the intended ranges. This uncertainty in  $T_e$  is a key reason for the Xe theta pinch experiment, and illustrates the fact that we are bootstrapping the design of the theta pinch experiment, since the unknown Xe atomic properties determine the conductive and radiative properties of the Xe plasma.

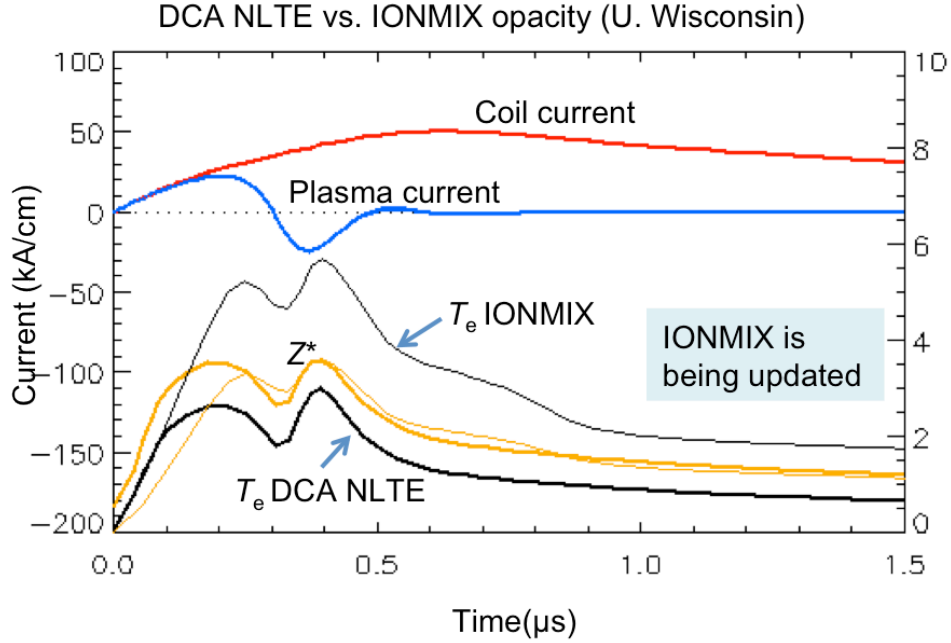


Figure 9. Different opacity models predict different temperatures. Such a temperature uncertainty is significant for LIFE target survival.

## 3. 3D ALE3D MODEL OF COIL STRESS

To confirm that the coil motion in response to inductively induced forces will not damage the tube, or the coil itself, coil stresses and displacements are assessed using the ALE3D MHD code, as shown in Figure 10. It is not clear that small displacements could be accurately captured on the complex mesh required to represent the 8-turn center-fed coil, so to begin with, the simpler two-turn center-fed coil with is modeled. Half of one of the half-coils is modeled (7 cm in  $z$ ), because there is an approximate symmetry plane in the center (in  $z$ ) of the half-coil; that symmetry end is held fixed spatially and the other end is free. The inside faces of the leads are fixed, because they will be attached to the rest of the circuit structure. The tube, the plasma, and the inductive feedback from the plasma to the tube are not modeled. The drive current source is applied to the ends of the leads from the circuit to the coil, and is implemented using a voltage source and a Thevenin resistor. Two drives are used:

1. the actual drive from the circuit model
2. the same drive but with the current held at the peak to investigate the approach to static stresses, to determine how well static models such as hoop stress describe the coil response.

The models are carried out to 40  $\mu\text{s}$ , long compared to the 0.5  $\mu\text{s}$  rise of the drive.

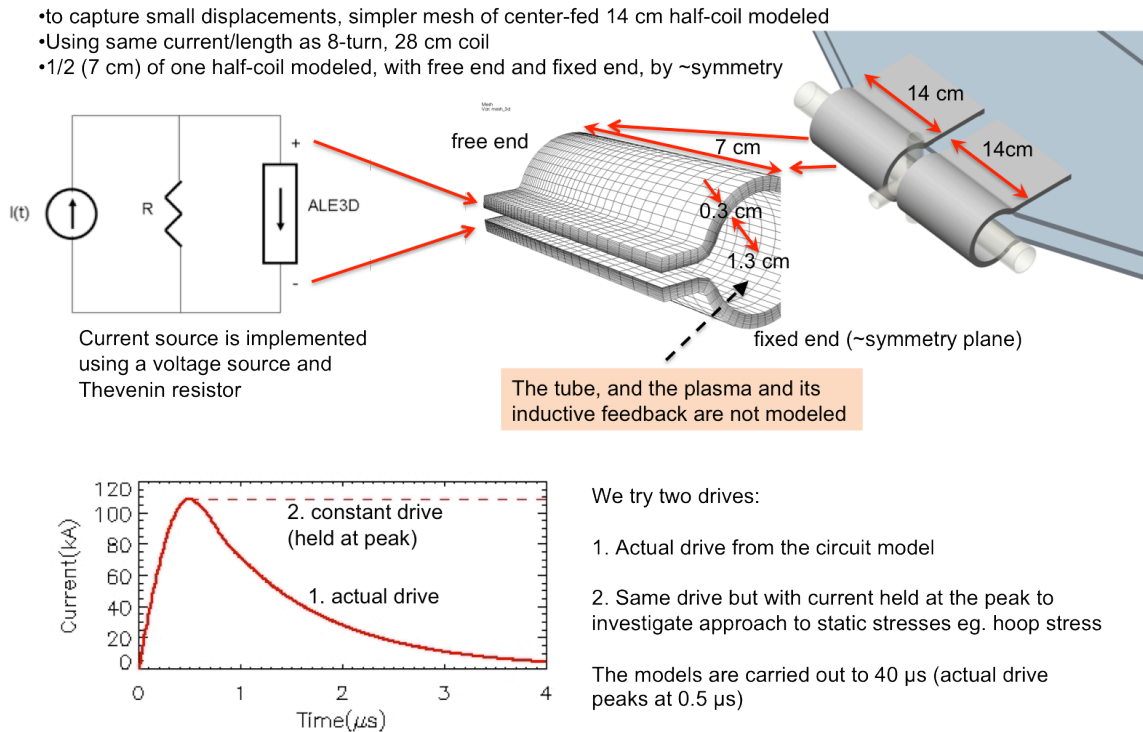
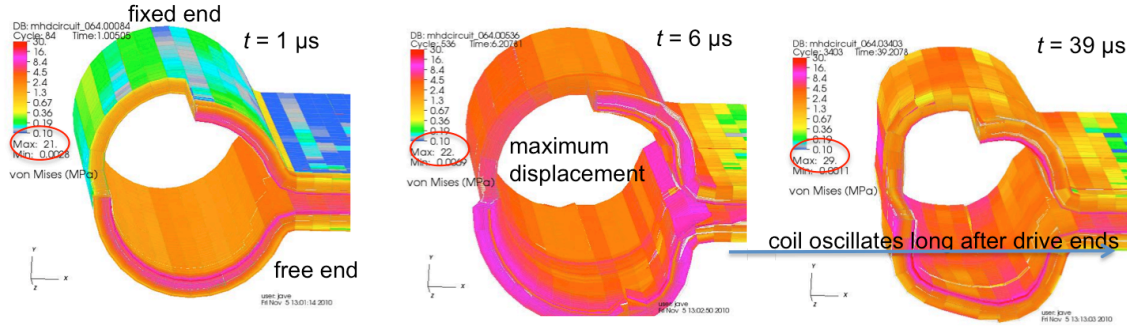


Figure 10. 3D coil stress are being assessed using the ALE3D MHD code.

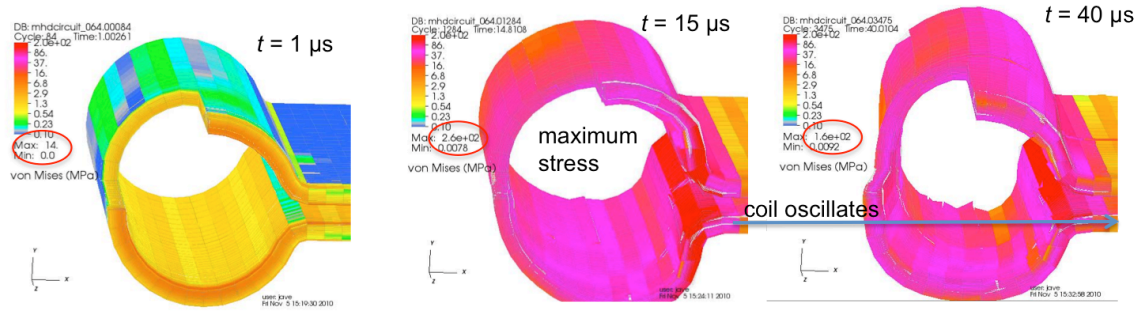
Figure 11 shows pseudocolor plots of von Mises stress on meshes with the displacements exaggerated by a factor of five thousand for the actual drive, and by two thousand for the constant drive. For the actual drive, a peak stress of about 30 MPa is reached, well below the 100 MPa level of concern, and the maximum displacement is less than three  $\mu\text{m}$ . For the constant drive, the peak stress exceeds 250 MPa and the displacements are below 15  $\mu\text{m}$ . The peak displacements occur at the free ends. Large stresses appear at the inside coil edge at the free end, and at the junction between the lead and the coil. In models where the leads allowed to move, the strong magnetic pressure between the leads accelerates them away from each other, so that the coil is also pried open. For both drives the coil continues to deform and oscillate out to 40  $\mu\text{s}$ . This suggests that the coil response is dynamic, and the stresses achieved are not well described by static models. This is reasonable, since at cold sound speeds of order 5 km/s (5 cm/ $\mu\text{s}$ ) in the coil, the drive is over long before reflected waves in the coil can make the coil respond as a unit.

Pseudocolor plots of von Mises stress on meshes with displacement exaggerated

1) Actual drive, exaggeration factor 5e3: 30 Mpa and  $< 3 \mu\text{m}$



2) Constant drive, exaggeration factor 5e2:  $> 250 \text{ Mpa}$  and  $< 15 \mu\text{m}$



dynamic response — static conditions not approached

Figure 11. von Mises coil stress is 30 Mpa; displacement is less than  $3 \mu\text{m}$ .

## 4. CONCLUSION

A theta pinch experiment intended to produce a Xe plasma at conditions relevant to the LIFE fusion chamber is being assessed with modeling. Plasma response is being assessed the HYDRA MHD radiation-hydrodynamics code with the DCA atomic physics model, in 1D cylindrical geometry. The key results so far are that  $T_e$  of 2-3 eV will be achieved before cooling, that the plasma response will be dynamic and complex, and that pinching will moderate the Joule heating. In addition, tube radius could be optimized to mitigate pinching and maximize Joule heating; a background  $B_{0z}$  field produced by fixed magnets or second coil could delay pinching; back EMF from the plasma coil is a 15–20% effect; the Hall term electric field is expected to produce charge separation; and different opacity models predict Xe ion temperatures sufficiently different to be significant for LIFE target survival.

Coil stress and displacement are being modeled using the 3D ALE3D material MHD code. ALE3D MHD modeling suggests that peak von Mises stress below 30 Mpa will be reached, well below the level of concern of 100 Mpa. The modeling also suggest that static stress conditions not approached, so that the coil response is not described well by static models such as hoop stress.

Issues being addressed in ongoing work include 2D end effects, plasma-specific effects (*eg.* the Hall term), 3D MHD instabilities, sensitivity to atomic models, and the cooling behavior out to 100 ms relevant to LIFE chamber conditions. Accurately modeling the late time cooling requires the data the theta pinch experiment is intended to provide.

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